

C. Stability of Shock Waves

The behavior of most materials subjected to shock conditions will satisfy the so-called Bethe-Weyl conditions.¹⁹ They are

$$(\partial P / \partial V)_S < 0 \quad (10)$$

$$(\partial^2 P / \partial V^2)_S > 0 \quad (11)$$

$$(\partial P / \partial S)_V > 0. \quad (12)$$

The first two conditions state the requirement that the shock velocity increase with pressure and the last condition states that the shock process is not isentropic; however, the shock process is assumed to be adiabatic. These are the necessary requisites for a shock wave to be stable, meaning, the wave will not separate into multiple waves nor exhibit dispersion. Dispersion would result if the Hugoniot curve in the P-V plane were concave downward.

There are two general properties of materials for which the second Bethe-Weyl condition is sometimes not satisfied, namely, elastic-plastic effects and phase changes. Because liquids have negligible rigidity, elastic-plastic effects are not observed and hence will not be discussed. Phase changes do occur in liquids and their effects will be discussed in terms of shock wave stability.

Consider the Hugoniot curve of Fig. 2 (a) in which a phase change occurs at P_1, V_1 . A shock wave which connects the state P_0, V_0 to any other state on the Hugoniot curve up to P_1, V_1 will be stable since the shock velocity increases with the pressure. The conservation relations expressed in Eqs. (7), (8), and (9) can be rearranged to provide an expression for the shock velocity in terms